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MODELLING CROWD DYNAMICS FROM A KINETIC THEORY VIEWPOINT

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Abstract

In this paper, we review a few aspect of complex living systems, the guidelines towards the study of crowd's dynamics and of pedestrians systems. This class of models belongs to complex systems in applied sciences, who interact in a non-linear manner and have a self-organizing ability. This follows the recipe of Boltzmann's kinetic gas theory, leading to the basic equations of gas dynamics in the limit.

1. Introduction and Motivation

The paper is concerned with the mathematical modelling of complex living systems. Let us consider a large system of interacting particles that belong to physical systems, whose dynamics is determined by their ability to dialogue among themselves and develop specific strategies. Typical examples are pedestrians in crowds, vehicles-driver on roads, or animals in swarms. If the strategy changes according with the number of

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interacting particles, all these systems can be classified as complex systems according to the observation that the dynamics of a few entities does not generate straightforwardly the dynamics of the whole system. Consequently, emerging collective behaviours do not appear to be straightforwardly related to the individual dynamics. Roughly speaking, complexity refers to the study of systems composed of many interacting components, or agents, that act together in a non-linear fashion and produce patterns of behaviour at the level of the group. In this review, we show how the mathematical approach can be applied in complex living systems.

We take the example of modelling of the crowd and we derive three categories of models. A crowd is a complex system and the collective behaviours of pedestrians in the crowd are often viewed as the emergent properties of the system. The first model we presented is formulated as a system of ODEs that describe the crowd behaviour such that it takes individual parameters like desired, velocity, actual velocity, and position. The second model is macroscopic (fluid) model, which is good enough to reproduce and evaluate generic crowd behaviour under different environmental conditions and the third model is a kinetic, Vlasov-type and Boltzmann-like equation based on interactions between the pedestrians. The concepts and techniques of statistical physics are being used nowadays to study several aspects of complex systems.

The questions which are of practical interest that we address can be summarized as follows:

- What are appropriate mathematical equations that govern complex living systems?

- In comparison with the flow traffic, can the analysis developed within the framework of the vehicular traffic be extended to the modelling of the human behaviour?

- What are the advantages and disadvantages of modelling?

We will focus on some aspects of theses questions.

2. Crowd Motion Modelling: A Brief Overview

As mentioned in the Introduction, typical examples of complex systems are crowd dynamics [29, 2], vehicular traffic [7], biological systems [13], consumers in the market [28], and social sciences [1]. Modelling crowd dynamics is quite recent and is mainly derived from vehicular traffic modelling, which has been widely analyzed in the field of applied mathematics and transportation engineering. Nevertheless, there is important difference between the dynamics of vehicles and that of the crowd. Because of the apparent similarities between these systems, the tools of statistical mechanics seem to be the natural choice for studying these models. There was a vast literature on modelling of the movement of crowds since the work of Hakin and Wright [17]. In this review, we will focus on the physics aspects of the modelling of crowd movement. The literature on crowd dynamics, using methods from physics is initiated by the works of Henderson [21, 22]. The author conjectured that pedestrian crowds behave similarly to gases and fluids. In practice, pedestrian crowd models, that is based on this conjecture, contain corrections due to interactions such as collision avoidance and deceleration maneuvers, which do not obey momentum and energy conservations [19].

Some papers are available mainly concerning modelling issues at the microscopic and macroscopic scale. Among others, Helbing and Molnar [19, 18]. Recent papers by Hughes [25, 26] and Coscia and Canavesio [9] which deal with macroscopic type modelling, put clearly in evidence that the modelling can be developed only if the thinking ability of interacting individuals is often carefully taken into account. In framework of scalar conservation laws, we can mention the reference [8]. The measure theoretical framework is proposed by Piccoli and Tosin [14]. An up-to-date review and critical analysis of crowd models so far proposed can be found in Bellomo and Dogbe [2].

Individuals in crowds can be regarded as complex living systems, who interact in a nonlinear manner. Moreover, interactions follow specific strategies generated by the ability to communicate with the other entities, and to organize the dynamics according both to their own strategy and interpretation of that of the others. Therefore, the knowledge of the interaction of a few entities is not sufficient to describe the collective dynamics of the overall system. A further difficulty is generated by the fact that individual dynamics are not generally observable, while only the overall behaviour can be observed and geometrically interpreted.

Crowd models can be classified into three categories, as already been done for vehicular traffic models, that correspond to different scales of the phenomenological observation of the system. Nowadays, one distinguishes three different conceptual frameworks for modelling crowd motion. The first is "microscopic" representation scale is explicitly focused on individual pedestrian each of which is represented by a "particle". The nature of the interactions among these particles is determined by the way the pedestrians influence each others' movement. The second is continuum (or macroscopic) representation scale which views pedestrians as a (continuous) fluid flow and suggests a focus on global crowd behaviour. In contrast, in the so-called "gas-kinetic" models, the traffic of pedestrians is viewed as a compressible fluid formed by the pedestrians. Once the representation scale has been chosen, it is useful to express all the variables involved in the problem in a dimensionless form. It is also well understood that none of the above representations is fully appropriate, considering that the number of interacting entities in crowds is not large enough to justify either the continuum mechanics approximation.

In order to provide a broad perspective, we describe each of the three modelling approaches including advantages and disadvantages of each.

Pedestrians in a crowd move in complex geometries generally in two or more space dimensions. From now, let us consider the system in two space dimension and let $\Omega \subset \mathbb{R}^2$ be the domain occupied by the crowd, that can be either bounded or unbounded (see Figure 1).

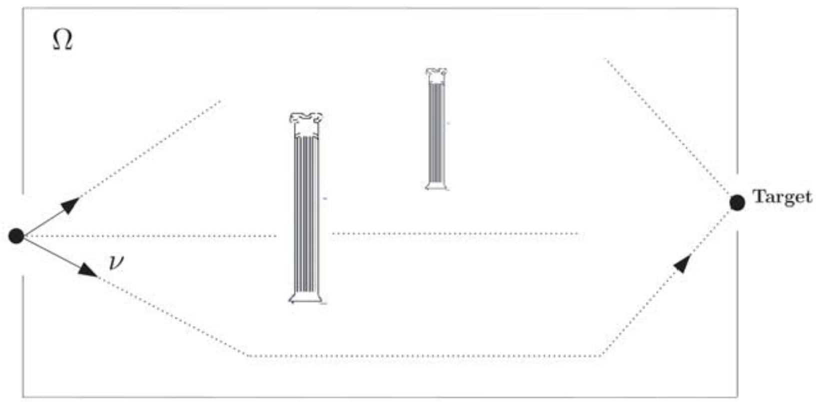

Figure 1. Overview of the geometry of crowd domain. Plan view of crowds moving through an environment containing a goal and an obstacle.

We provide a brief (incomplete) overview of the classification into three categories of models for the movement of the crowd.

2.1. Microscopic models

Microscopic description refers to entities individually identified (e.g., [15]). In this case, their position and velocity identify, as dependent variables of time, the state of the whole system. Models developed at the microscopic scale are stated in terms of ordinary differential equations. Then, similar to the Newtonian mechanics for systems of particles, one has to solve a large system of equations. Mean quantities, such as density and mass velocity, are then obtained by an averaging process. Bearing all above in mind, the overall description of microscopic models is delivered by the following system of equations:

$$\begin{cases} \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \\ \dfrac{d\mathbf{v}_i}{dt} = \mathbf{F}_i(\mathbf{x}_i, \ldots, \mathbf{x}_N, \mathbf{v}_i, \ldots, \mathbf{v}_N), \end{cases} \tag{2.1}$$

where $\mathbf{x}_i \equiv \mathbf{x}_i(t) = (x_i(t), y_i(t))$ is the dimensionless position vector in Ω and $\mathbf{v}_i \equiv \mathbf{v}_i(t) = (v_x^i(t), v_y^i(t))$ is the dimensionless velocity of each i-th velocity for each i-th pedestrian with $i \in \{1, \ldots, N\}$. The forces $\mathbf{F}_i(t)$ involved including acceleration and deceleration due the various reactions of the individuals when they perceive their environment (other individuals and obstacles). $\mathbf{F}_i(t)$ is the sum of forces acting on the individual i at time t.

Microscopic models include the cellular automaton model (e.g., [5, 6]), the lattice gas model (e.g., [16]), magnetic force models (e.g., [30]) and are particularly well suited for use with small crowds. One can also add to these models, the optimal control approach given by Hoogendoorn and Bovy [24] and Lagrangian formalism by rigid disks introduced by Maury and Venel [29].

The most popular microscopic models of pedestrians is the social force model proposed by Helbing and Molnár (see [18] and the references therein), which is a self-driven, many-particles model using push-pull effects to describe pedestrian behaviour in crowds. However, the idea of social force models is modelling the behaviour using only a set of simple forces to describe the behaviour of the human pedestrians that comprise a crowd. The resulting force for pedestrian i, $\mathbf{F}_i$ is the sum of the three main forces

$$\mathbf{F}_i(\mathbf{x}_i, \mathbf{v}_i) = F_i^{\mathrm{drv}}(t) + \sum_{\substack{j=1 \\ i \neq j}}^{N} F_{ij}(\mathbf{x}_i, \mathbf{x}_j) + \sum_{k=1}^{N_w} F_{ik}(\mathbf{x}_i) + \xi_i(t). \tag{2.2}$$

The first term in the right hand side of (2.2) represents the driving forces, i.e., the force to drive pedestrians to a desired direction with a desired speed to reach the target. The standard form of the driving force is

$$F_i^{\mathrm{drv}}(t) = \frac{1}{\tau}(\mathbf{v}_i^0(t)e_i - \mathbf{v}_i(t)). \tag{2.3}$$

In Equation (2.3), $\mathbf{v}_i^0$ is the intended velocity with which pedestrians tend to move in the absence of interaction; e_i is the unit vector pointing towards the pedestrian's target; v_i is the actual velocity of the pedestrian at time t. τ is the relaxation time, and scales the force, determining how fast a pedestrian changes velocity and returns to its desired velocity after having been walking slower because of obstacles etc.

The second term in the right-hand side of Equation (2.2) represents the interaction force from pedestrians j. The third term in the right-hand side of Equation (2.2) is the interaction potential between pairs of pedestrians, which can takes, for example, the form

$$F_{ik}(\mathbf{x}_i) = -\nabla U(\|r_{ik}\|), \tag{2.4}$$

where U is a monotonic decreasing potential and r_{ik} denotes the shortest distance between pedestrian and wall or obstacle; N_w is the number of obstacles and walls. The last term in the right-hand side of Equation (2.2), $\xi_i(t)$ represents a fluctuation term that stands for random behavioural variations arising from accidental or deliberate deviations from the optimal strategy of motion.

A primary advantage of microscopic models is the ability to study individual pedestrian motion. The main disadvantage of microscopic models, is that one ordinary differential equation is required for each pedestrian. In addition, microscopic models become very expensive simulations of large systems of equations.

2.2. Macroscopic models

The approach at the macroscopic scale has been settled by Hughes [25], and subsequently developed by various authors [2, 29, 32], by means of classical methods of continuum mechanics based on the use of mass and momentum conservation equations properly closed by phenomenological models modelling the relation of the acceleration term, or mean velocity, to local flow conditions. Hughes' model is revisited by [27]. Specifically, the mathematical framework is identified by mass and momentum conservation equations, namely, the trajectories of the individuals are found by solving a coupled partial differential equation system consisting of equation of motions for each agent.

The general mathematical framework is given by the three partial differential equations (PDEs), expressing the conservation of mass, momentum and energy

$$\begin{cases} \partial_t \rho + \nabla_{\mathbf{x}}(\rho \mathbf{v}) = 0, \\ \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \mathbf{v} = \mathbf{A}[\rho, \mathbf{v}]. \end{cases} \tag{2.5}$$

Here, $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ is the dimensionless velocity, $\mathbf{x} = (x, y)$ represents coordinates, $\mathbf{A}[\rho, \mathbf{v}]$ models the component of the mean acceleration, acting on pedestrians. The square brackets indicate that it may be a functional of its arguments.

According to the specific constitutive assumption, different models can be derived that involve only some of the three equations in (2.5). They can be classified as follows:

- Scalar or first order models: They are described by mass conservation equation only, and by a closure equation $\mathbf{v} = \mathbf{v}[\rho]$ that links the local velocity to the crowd density (e.g., Hughes [25]).

- Second order models: They are obtained by mass and linear momentum conservation equations with the addition of a phenomenological relation describing the psycho-mechanic action $\mathbf{A}[\rho, \mathbf{v}]$ on the pedestrians.

- Higher order models: They use all the equations in (2.5), with a suitably defined energy density.

The presented classification corresponds to increasing accuracy in the description of crowd dynamics, but also to increasing complexity. In particular, higher order models introduce more parameters to be identified and therefore, they are very difficult to be handed and compared with the experimental observation. Therefore, in the following, more attention will be paid to the description of first order models.

The following hypothesis about the nature of crowd motion were adopted to obtain Equation (2.5):

Assumption 1.

Pedestrians seek to minimize their (accurately) estimated travel time but temper their velocity according to local density. Specifically, in each point of the domain, individuals move towards a given objective along the direction $\nu(x, y)$.

Formally, the acceleration term writes

$$\mathbf{A}[\rho, \mathbf{v}] \equiv \psi[\rho, \mathbf{v}]\nu,$$

where ν is the unit vector representing the direction of pedestrian towards the target, and ψ is determined according to the type of the models. The general form of ψ is

$$\begin{cases} \psi[\rho, \mathbf{v}] \equiv \psi(\rho, \nabla_{\mathbf{x}}, \mathbf{v}), \\ \nu = \nu[\rho]. \end{cases}$$

According to the specific constitutive assumption, different models can be derived that involve only some of the two equations in (2.5). One example for this class is the model by Hughes [25], which is motivated by considering the crowd as a ‘thinking fluid’. In this class of model, only the continuity equation of (2.5) is used, that is, the first equation of (2.5), the so-called scalar or first-order models is obtained, and the closure relations for the velocity $\mathbf{v}$ is a term of the density ρ and possibly also of its gradients $\nabla\rho$, that is (e.g., [9])

$$\mathbf{v}[\rho, \nabla\varrho](x) = \psi[\rho, \nabla\rho]\nu(x).$$

The most popular macroscopic model of first-order is proposed by Hughes [25, 26]. Writing the density of pedestrians or the density of holes, as appropriate, as ρ, the governing equations for the flow are given as

$$\frac{\partial\rho}{\partial t} - \nabla \cdot (\rho g(\rho) f^2(\rho)\nabla\phi) = 0, \tag{2.6}$$

where in (2.6), the pedestrian velocity is given by

$$\mathbf{v} = -g(\rho)f^2(\rho)\nabla\phi, \tag{2.7}$$

with ϕ denotes the potential field giving the direction of motion toward a common destination, g is discomfort function at high density, f is speed diagram. In the second-order model, the acceleration in (2.5) consists of two contributions: The first one corresponding to a trend and to equilibrium velocity depending on the local density, directed towards ν, and the second term to the action of the density gradients towards ν. In particular, negative values increase the acceleration, while positive values decrease it. The reader will have in mind that, one speaks about acceleration and not about force because it would not be adapted for a system in which can not be defined properly the mass. To be clear: The acceleration in here is not the real physical force that has dimension of Newton but only the analogy of the force that characterizes the internal driving force or motivation of the pedestrian. The same type of model is used in [12] to study the behaviour of a flock of sheep. In a companion paper [2], we have developed different models of the system of Equations (2.5).

On the modelling approach of macroscopic models include among others, the optimal control approach by Maury et al. [32] and the mean-field games approach by Dogbe [10]. One advantage of macroscopic models is that they are relatively simple in terms of calculations, compared to microscopic models. These models have fewer parameters than their microscopic. Meanwhile, one of the disadvantages of a macroscopic model is the loss of small detail, the dynamics that can be modelled with microscopic models.

2.3. Statistical or gas-kinetic models

In the kinetic theory, pedestrian traffic is treated as a gas of interacting particles where each particle represents a pedestrian. The probabilistic description of pedestrian crowds in the kinetic theory is developed by appropriately modifying the kinetic theory of gases. Thus, statistical models consist of the derivation of an evolution equation for the distribution function on the position and velocity of the pedestrians along the walkway. This approach was first applied by Henderson in [22], who has showed that the movements of people in crowds seem to obey the Maxwell-Boltzmann statistics of the kinetic theory of gases. The ideas presented were subsequently extended by the authors [19, 23]. The kinetic theory description is used when the state of the system is still identified by position and velocity of the microscopic entities, however, their representation is delivered by a suitable probability distribution over the microscopic state. Mathematical models describe the evolution of the above distribution function generally by nonlinear integro-differential equations. The representation is defined by the statistical distribution of their position and velocity

$$f = f(t, \mathbf{x}, \mathbf{v}), \quad [0, T] \times \Omega \times D_{\mathbf{v}} \to \mathbb{R}^+. \tag{2.8}$$

Here, $D_{\mathbf{v}} \subset \mathbb{R}^2$ is the domain of the velocity variable. If f is locally integrable, $f(t, \mathbf{x}, \mathbf{v})d\mathbf{x}d\mathbf{v}$ denotes the number of individuals, which, at the time t, are in the elementary domain of the microscopic states $\Omega \times D_{\mathbf{v}}$. Then, macroscopic observable quantities can be obtained, under suitable integrability assumptions, by moments of the distribution. In particular, the dimensionless local density is given by

$$\rho(t, \mathbf{x}) = \int_{D_{\mathbf{v}}} f(t, \mathbf{x}, \mathbf{v}) d\mathbf{v}.$$

The total number of individuals in the closed domain Ω, occupied by the crowd at the time t, is given by

$$N(t) = \int_{\Omega} \rho(t, \mathbf{x}) d\mathbf{x},$$

which depends on time in the presence of inlet and/or outlet of pedestrians. The local flux is defined as

$$\mathbf{q}(t, \mathbf{x}) = \int_{D_{\mathbf{v}}} \mathbf{v} f(t, \mathbf{x}, \mathbf{v}) d\mathbf{v}.$$

Analogously, the mean velocity can be computed as follows:

$$E[\mathbf{v}](t, \mathbf{x}) = \frac{1}{\rho(t, \mathbf{x})} \int_{\Omega} \mathbf{v} f(t, \mathbf{x}, \mathbf{v}) d\mathbf{x},$$

and similarly the speed variance, provides a measure of the stochastic behaviour of the system with respect to the deterministic macroscopic description. We now formulate the kinetic description of the above motion laws. In the presence of external accelerations and neglecting long-range interactions, the number (or mass) phase space density of a pedestrian undergoing collisional events, satisfies a kinetic equation

$$\frac{\partial}{\partial t} f(t, \mathbf{x}, \mathbf{v}) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f(t, \mathbf{x}, \mathbf{v}) + \nabla_{\mathbf{v}} \cdot \big((\mathbf{F}[\rho_f] + \mathcal{F}[f]) f(t, \mathbf{x}, \mathbf{v}) \big) = 0. \quad (2.9)$$

Here $a \cdot b$ denotes the scalar product of two vectors a and b in $\mathbb{R}^2$ and $\nabla_{\mathbf{v}}$ stands for the divergence operator with respect to the velocity variable $\mathbf{v} \in \mathbb{R}^2$. Furthermore, $f = f(t, \mathbf{x}, \mathbf{v})$ is given by summing all actions applied by pedestrians; $\mathbf{F}[\rho_f]$ is the macroscopic acceleration and $\mathcal{F}$ denotes the microscopic acceleration. This equation bears deep similarity to the Boltzmann equation of statistical physics. Naturally, pedestrian cannot observe f itself, but some lower order moments of f, like the mean velocity, the mean acceleration … etc. The modelling problem consists then in the mathematical description of the two accelerations, respectively, depending on various conceivable physical situations. One can add to the Equation (2.9) a nonlocal operator $\mathcal{J}[f]$ of local interactions (typically an integral operator) depending on the local distribution function. Thus, Equation (2.9) can be rewritten as

$$\frac{\partial}{\partial t} f(t, \mathbf{x}, \mathbf{v}) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f(t, \mathbf{x}, \mathbf{v}) + \nabla_{\mathbf{v}} \cdot ((\mathbf{F}[\rho_f] + \mathcal{F}[f]) f(t, \mathbf{x}, \mathbf{v}))$$

$$= \mathcal{J}[f, \rho; \nu](t, \mathbf{x}, \mathbf{v}). \tag{2.10}$$

The operators $\nabla_{\mathbf{x}}$ and $\nabla_{\mathbf{v}}$ are the gradient and the divergence of vector fields or tensor. The left-hand side is a transport operator. It expresses the material derivative of f in the phase space spanned by $(\mathbf{x}, \mathbf{v})$, due to the motion of the particles with velocity $\mathbf{v}$ and the "force terms" $\mathbf{F}[\rho_f] + \mathcal{F}$. The modelling problem consists then in the mathematical description of the term $\mathcal{J}[f, \rho; \nu]$ depending on various conceivable physical situations. Of course, Equation (2.10) is incomplete until we specify the term $\mathcal{J}[f, \rho; \nu]$. In general, one way to model the term $\mathcal{J}$ consists in describing a trend to equilibrium analogous to the BGK Boltzmann model in kinetic theory (see [19, 11]):

$$\mathcal{J}[f, \rho, \nu](t, \mathbf{x}, \mathbf{v}) = \omega(\rho)(f_e(\mathbf{v}|\rho, \mathbf{v}) - f(t, \mathbf{x}, \mathbf{v})), \tag{2.11}$$

where the rate of convergence $\omega(\rho)$ depends on the local density ρ, and f_e denotes the equilibrium distribution function that may be parameterized by the local density ρ and by the direction ν towards the target. We recall that the BGK equation is a model kinetic collisional equation, which can be considered in any dimension $d \geqslant 1$ and which takes into account only the global effect of interactions between fluid particles: such an effect is expected to be a relaxation towards local thermodynamic equilibrium.

In Equation (2.10), when the interaction term J is vanished, $f \equiv f(t, \mathbf{x}, \mathbf{v})$ can be rigorously proved to satisfy the following time-continuous stochastic model for the pedestrian positions $\mathbf{x}$ and velocity directions $\mathbf{v}$ (see, e.g., [4]):

$$\frac{\partial}{\partial t} f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \nabla_{\mathbf{v}} \cdot ((\mathbf{F}[\rho_f] + \mathcal{F}[f]) f) = \sigma \Delta_{\mathbf{v}} f. \tag{2.12}$$

Here, $\Delta_{\mathbf{v}}$ stands for the Laplace operator with respect to the velocity variable $\mathbf{v} \in \mathbb{R}^2$. The right-hand side is a velocity diffusion term which comes from the velocity noise. This is an example of the dynamics of crowds in panic or evacuation situations.

In fact, the description of the system by methods of mathematical kinetic theory means to define first of all the microscopic state of entities interacting in a large system formed by these entities, and the distribution function on this state. The microscopic state always includes geometrical variables suitable to identify their position and form, as well as their mechanical quantities according to their speed. However, in the case of living systems, the identification of the microscopic state requests an additional variable, called "activity", which is characteristic of a modelled particular system. For example, this variable may be related to the social state in the case of dynamics of populations. Thus, the study of these models can be pursued by suitable development of the so-called mathematical kinetic theory for active particles, which has shown to be a useful reference applications in other fields of life sciences. Traditionally, methods of mathematical kinetic theory have been applied to model the evolution of large systems of interacting classical or quantum particles. Recently, the collective behaviour of a large population of interacting individuals has been recently studied by using methods of mathematical kinetic theory.

The mathematical problem to face is the derivation of an evolution equation for the one particle distribution function over the microscopic state of the active particles. The literature of kinetic theory can be found in the review paper Perthame [31], while mathematical structures concerning the kinetic theory of active particles are proposed in the book [33]. The reasonings proposed in the preceding sections can be almost straightforwardly extended to crowd dynamics. However, additional difficulties have been carefully considered, although the mathematical structures to be used for the modelling are technically the same with the simple modification of the addition of further space variables.

Roughly speaking, if one introduces an activity variable, denoted $u \in \mathbb{R}$, in the microscopic state of pedestrians to express their strategy, and now denotes, for all $t \geqslant 0$ by $f(t, \mathbf{x}, \mathbf{v}, u) d\mathbf{x} d\mathbf{v} du$ the number of active pedestrians whose state, at time t, is in the elementary volume $[\mathbf{x}, \mathbf{x} + d\mathbf{x}] \times [\mathbf{v}, \mathbf{v} + d\mathbf{v}] \times [u, u + du]$, then the equation of motion of crowd takes the form

$$\frac{\partial f}{\partial t}(t, \mathbf{x}, \mathbf{v}, u) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f(t, \mathbf{x}, \mathbf{v}, u) + \nabla_{\mathbf{v}} \cdot (\mathscr{F}[f] f(t, \mathbf{x}, \mathbf{v}, u)) = J[f], \tag{2.13}$$

where $\nabla_{\mathbf{v}} \cdot (\mathscr{F}[f] f)$ is the acceleration term that represents the interaction between individuals when they are distant. Recently, to take into account, the strong granular nature of the flow of pedestrians some models have been proposed in the literature, based on the discrete kinetic theory [3].

The major advantage of this approach is due to the large variety of kinetic models, which one can obtain in the capture of the non-equilibrium physical phenomena for the microscale gas flow simulation. This approach allows to connect all macroscopic flow variables on a single particle distribution. Furthermore, this approach can be considered as being closer to the real physical phenomena.

The main disadvantage of gas-kinetic models of pedestrians is that, looking for a numerical solution is very difficult to obtain. In addition, a realistic gas-kinetic or fluid-dynamic theory for pedestrians must contain corrections due to their particular interactions (that is, avoidance and deceleration maneuvers) since momentum and energy are not conserved in pedestrian motion [19].

3. Conclusion

In this survey, where many interesting aspects had to be left out, we have presented a general framework to model living complex systems, through the example of the dynamics of the crowd, in the spirit of fluid theory. The analysis is developed in view of specific applications. The mathematical frameworks claim to be more general than those available in the literature; that, at least in principle, should allow one to include in the modelling process of certain physical systems additional descriptive ability with respect to the existing models. We believe that the presentation is very brief. The reader interested in this aspect of modelling crowd motion could refer to paper [2, 3].

Let us remark that a variety of interesting and challenging mathematical problems are related to the qualitative and computational analysis of problems generated by living systems. The are several issues that are left for future work. We hope that this brief overview has convinced the reader that the investigation of the motion of crowd is a fascinating field, both for its practical relevance and the insights into the physics of systems far from equilibrium.

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